Scattering Analysis of a Coaxial Line Terminated by a Gap

Hyo J. Eom, Young C. Noh, and Jong K. Park

***Abstract*—TEM-wave scattering on a coaxial line terminated by a gap is theoretically investigated. The Fourier transform and the mode matching are used to obtain a rapidly convergent series solution. Our solution is compared with Marcuvitz result based on the small-aperture method. Computations are performed to illustrate the phase behavior of reflection versus a gap variation.**

***Index Terms*—Coaxial-cable discontinuity, Fourier transforms, mode-matching, reflection coefficient.**

I. Introduction

A PROBLEM of TEM-wave scattering on a coaxial line terminated by a gap (see Fig. 1) has been of fundamental interest in microwave engineering. Marcuvitz [1] first obtained an approximate solution based on the small-aperture method, and the numerical solution [2] was compared with Marcuvitz solution. A similar problem of coaxial re-entrant cavity with partially dielectric filled gap was considered in [3] for its application for permittivity measurements. In this letter we obtain a new, rigorous solution for scattering on a coaxial line terminated by a gap, using the Fourier transform and the mode matching as used in [4]. In the next section, we present a closed-form solution in rapidly convergent series which are numerically efficient. The notations and scattering analysis in the next section closely follow those in [4].

II. Scattering Analysis

Fig. 1 shows a problem of TEM-wave reflection on a coaxial line terminated by a gap. Note that the inner conductor of a coaxial line is partially removed and replaced by a dielectric with permittivity $\epsilon_2 = \epsilon_0\epsilon_{r2}$. A time factor $e^{-i\omega t}$ is suppressed throughout. Let a TEM wave $[H_\phi^i = e^{ik_1 z}/(\eta_1\rho)]$ be incident from left inside the coaxial line in Region I, then the scattered H-field is

$$H_{\phi \mathrm{I}}(\rho, z) = e^{-ik_1 z}/(\eta_1\rho) + i\omega\epsilon_1 \frac{2}{\pi}\int_0^\infty \cdot \frac{1}{\kappa}\tilde{E}_I(\zeta)R'(\kappa\rho)\cos(\zeta z)\,d\zeta \tag{1}$$

where $\kappa = \sqrt{k_1^2 - \zeta^2}$, $k_1 = \omega\sqrt{\mu\epsilon_1} = 2\pi/\lambda_1$, $\eta_1 = \sqrt{\mu/\epsilon_1}$, $R(\kappa\rho) = J_0(\kappa\rho)N_0(\kappa a) - N_0(\kappa\rho)J_0(\kappa a)$, and $R'(\cdot) = dR(\cdot)/d(\cdot)$. $J_0(\cdot)$ is the zeroth-order Bessel function and $N_0(\cdot)$ is the zeroth-order Neumann function. $\tilde{E}_I(\zeta)R(\kappa b)$ is the cosine-Fourier transform of $E_{zI}(b, z)$ given by $\tilde{E}_I(\zeta)R(\kappa b) = \int_0^\infty E_{zI}(b, z)\cos(\zeta z)\,dz$. In Region II, the field is

$$H_{\phi \mathrm{II}}(\rho, z) = i\omega\epsilon_2 \sum_{m=0}^\infty \frac{1}{\kappa_m} R_0'(\kappa_m\rho)\cos(a_m z) \tag{2}$$

where $R_0(\kappa_m\rho) = p_m J_0(\kappa_m\rho)$, p_m is an unknown coefficient to be determined by the boundary conditions, $\kappa_m = \sqrt{k_2^2 - a_m^2}$, $k_2 = \omega\sqrt{\mu\epsilon_2} = 2\pi/\lambda_2$, and $a_m = m\pi/d$. Applying the Fourier cosine transform to the E_z-field continuity

$$E_{z\mathrm{I}}(b, z) = \begin{cases} E_{z\mathrm{II}}(b, z), & -d < z < 0 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

yields

$$\tilde{E}_{\mathrm{I}}(\zeta) = \sum_{m=0}^\infty \frac{R_0(\kappa_m b)}{R(\kappa b)} F_m(\zeta) \tag{4}$$

where

$$F_m(\zeta) = \frac{(-1)^m\zeta[e^{i\zeta d} - e^{-i\zeta d}]}{2i(\zeta^2 - a_m^2)}. \tag{5}$$

The H_ϕ-field continuity at $\rho = b$ gives for $-d < z < 0$

$$H_{\phi\mathrm{I}}(b, z) + \frac{e^{ik_1 z}}{\eta_1 b} = H_{\phi\mathrm{II}}(b, z). \tag{6}$$

Multiplying (6) by $\cos(a_s z)$ (where $s = 0, 1, 2, \ldots$) and integrating with respect to z from $-d$ to zero, we obtain

$$\sum_{m=0}^\infty \left[J_0(\kappa_m b)I_{ms} + \frac{\alpha_m d\epsilon_{r2}}{2\kappa_m\epsilon_{r1}}J_1(\kappa_m b)\delta_{ms}\right]p_m = \frac{2i}{k_1 b}F_s(k_1) \tag{7}$$

where δ_{ms} is the Kronecker delta $\alpha_m = 2\,(m = 0),\ 1\,(m = 1, 2, \ldots)$, and

$$I_{ms} = \frac{2}{\pi}\int_0^\infty \frac{R'(\kappa b)}{\kappa R(\kappa b)} F_m(\zeta)F_s(\zeta)\,d\zeta \tag{8}$$

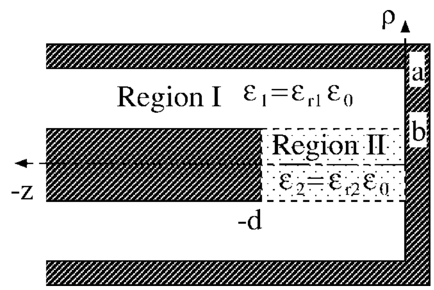

Fig. 1. The geometry of a coaxial line terminated by a gap.

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which is transformed into fast-converging series

$$I_{ms} = \frac{\alpha_m d R'(\kappa b)}{2\kappa R(\kappa b)} \delta_{ms}\Big|_{\zeta=a_m} - \frac{i(-1)^{m+s} k_1 (1-e^{2ik_1 d})}{2b \ln\left(\frac{b}{a}\right)(k_1^2 - a_m^2)(k_1^2 - a_s^2)} - \sum_{n=1}^{\infty} \frac{i(-1)^{m+s}\zeta(1-e^{2i\zeta d})}{b\left[1 - \frac{J_0^2(\kappa b)}{J_0^2(\kappa a)}\right](\zeta^2 - a_m^2)(\zeta^2 - a_s^2)}\Bigg|_{\zeta=\zeta_n} \quad (9)$$

where ζ_n is given by $R(\kappa b)|_{\zeta=\zeta_n} = 0$. The scattered field at $z < -d$ in Region I is

$$H_{\phi \mathrm{I}}(\rho, z) = \frac{(1+L_0)e^{-ik_1 z}}{\eta_1 \rho} - \sum_{n=1}^{\infty} L_n(\zeta) R'(\kappa\rho) e^{i\zeta z}\big|_{\zeta=-\zeta_n} \quad (10)$$

where

$$L_0 = \frac{k_1 \sin(k_1 d)}{\ln(b/a)} \sum_{m=0}^{\infty} \frac{(-1)^m R_0(\kappa_m b)}{k_1^2 - a_m^2} \quad (11)$$

$$L_n(\zeta) = \frac{2\omega\epsilon_1 \sin(\zeta d)}{bR'(\kappa b)\left[1 - \frac{J_0^2(\kappa b)}{J_0^2(\kappa a)}\right]} \sum_{m=0}^{\infty} \frac{(-1)^m R_0(\kappa_m b)}{\zeta^2 - a_m^2}. \quad (12)$$

The reflection coefficient Γ at $z = -d$ is

$$\Gamma = -\frac{H_{\phi \mathrm{I}}}{H_\phi^i}\Bigg|_{z=-d} = -(1+L_0)e^{i2k_1 d}. \quad (13)$$

In the low-frequency regime $(d \ll \lambda_1)$, p_0 becomes dominant, and $|p_m/p_0| \approx 0\,(m \geq 1)$. Hence, a dominant-mode solution is

$$p_0 \approx \frac{i2\sin(k_1 d)}{k_1^2 d\left[J_0(k_2 b) I_{00} + \frac{d\epsilon_{r2}}{k_2 \epsilon_{r1}} J_1(k_2 b)\right]} \quad (14)$$

$$\Gamma \approx -\left[1 + \frac{\sin(k_1 d) J_0(k_2 b)}{\ln(b/a) k_1} p_0\right] e^{i2k_1 d}. \quad (15)$$

Fig. 2 shows the comparison of the phase of Γ between our result and [1] when $d = 0.057$ cm, $a = 0.714$ cm, $b = 0.310$ cm, and $\epsilon_{r1} = \epsilon_{r2} = 2.1$, thus confirming an excellent agreement below 3 GHz. Note that the low-frequency solution (15) also yields a result as accurate as (13). Fig. 3 illustrates the behavior of the phase variation of Γ versus the gap distance for various parameters. The reference curve shows the phase of Γ when frequency = 2 GHz, $a = 0.714$ cm, $b = 0.310$ cm, and $\epsilon_{r1} = \epsilon_{r2} = 2.1$. Our computations show that an increase in ϵ_{r2} results in an increase in the phase of Γ. When either the frequency increases to 5 GHz or the outer radius a increases to 1.016 cm, the magnitudes of phase of Γ are seen to increase. Note that the phase of Γ changes drastically when $d < 0.1$ cm. Also note that the phase of Γ changes very little as d increases, because the capacitance between the inner conductor and the short-circuited termination at $z = 0$ becomes negligible for large d.

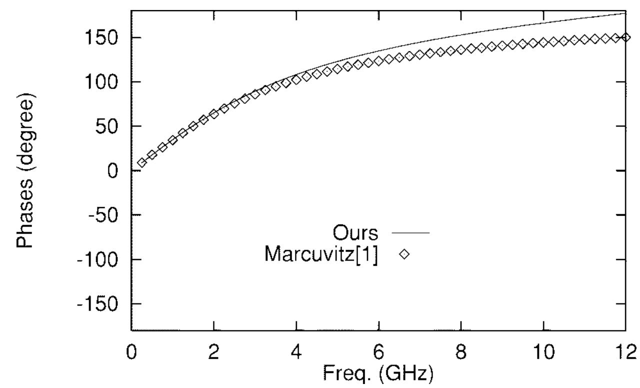

Fig. 2. Phase variation of the reflection, Γ versus the frequency when $d = 0.057$ cm, $a = 0.714$ cm, $b = 0.310$ cm, and $\epsilon_{r1} = \epsilon_{r2} = 2.1$.

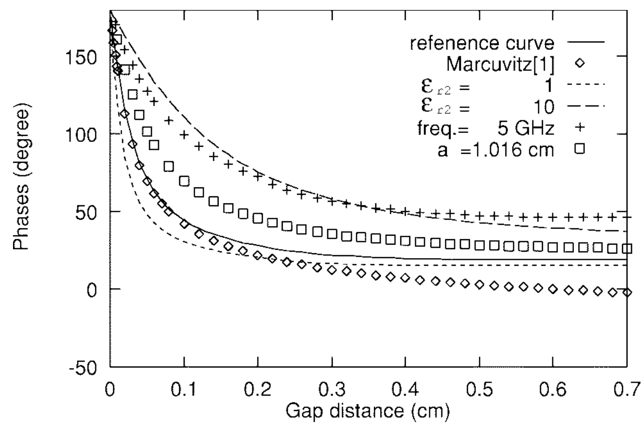

Fig. 3. Phase variation of the reflection, Γ versus the gap distance. The parameters of the reference curve are the frequency = 2 GHz, $a = 0.714$ cm, $b = 0.310$ cm, and $\epsilon_{r1} = \epsilon_{r2} = 2.1$. Unless specified, the parameters for other curves are the same as those of the reference curve.

III. Conclusion

A closed-form solution for reflection on a coaxial-line terminated by a gap is obtained in rapidly converging series. Numerical computations are performed to show the behavior of phase variation of reflection versus a gap variation.

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